Indian Statistical Institute
B.Math.(Hons.) III Year
Second Semester 2006-07
Mid Semester Examination
Algebraic Geometry

Time: 3 hrs Date:07-03-07 Total Marks: 50

Attempt all questions.

Assume $k$ is an algebraically closed field for all questions below.

1. Let $J = (xy + yz + xz,\ xyz) \subseteq k[x, y, z]$ and $X = V(J) \subseteq \mathbb{A}^3_k$. Describe the irreducible components of $X$. [6]
2. Let $f \in k[x_1, \ldots, x_n]$ and $X = V(f) \subseteq \mathbb{A}^n_k$. Describe $I(X)$ (in terms of the irreducible factors of $f$) and justify your answer. [7]
3. Give an example of two affine varieties $X, Y \subseteq \mathbb{A}^2_k$ such that
$$I(X \cap Y) \neq I(X) + I(Y).$$
[7]
4. Let $Y = V(x^7 - y^3) \subseteq \mathbb{A}^2_k$ and $\theta : \mathbb{A}^1_k \to Y$ be the morphism $t \mapsto (t^3, t^7)$. Check whether $\theta$ is a bijection, a homeomorphism, an isomorphism. [10]
5. Show that $\mathbb{A}^1_k$ is not isomorphic to any proper open subset of itself. [10]
6. Show that a regular function on $\mathbb{P}^1_k$ must be a constant, i.e., an element of $k$. [10]